Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Coefficient of Relative Risk Aversion

Suppose a household has preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \]

\[ u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta l_t \]

What is the household’s coefficient of relative risk aversion?
Coefficient of Relative Risk Aversion

Suppose a household has preferences:

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What is the household’s coefficient of relative risk aversion?

Answer: 0
Coefficient of Relative Risk Aversion

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$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\chi}}{1+\chi}$$

What is the household’s coefficient of relative risk aversion?

Answer: \(\frac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}\)
Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):
- Individuals who win a lottery prize reduce labor supply by $.11 for every $1 won (note: spouse may also reduce labor supply)

Coile and Levine (2009):
- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Coronado and Perozek (2003):
- Individuals who held more stocks in late 1990s retired 7 months earlier

Large literature estimating wealth effects on labor supply (e.g., Pencavel 1986)
Frictional Labor Markets

Perfectly rigid labor market:
- Arrow (1964), Pratt (1965), Epstein-Zin (1989), etc.

Perfectly flexible labor market:
- Swanson (2012, 2013)

This paper:
- Frictional labor markets
A Household

Household preferences:

\[ E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_{\tau}) - V(l_{\tau} + u_{\tau})], \]

Flow budget constraint:

\[ a_{\tau+1} = (1 + r_{\tau})a_{\tau} + w_{\tau}l_{\tau} + d_{\tau} - c_{\tau}, \]

No-Ponzi condition:

\[ \lim_{T \to \infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{\tau+1} \geq 0, \]

\{w_{\tau}, r_{\tau}, d_{\tau}\} are exogenous processes, governed by \(\Theta_{\tau}\)

Labor market search:

\[ l_{\tau+1} = (1-s)l_{\tau} + f(\Theta_{\tau})u_{\tau} \]
The Value Function

State variables of the household’s problem are \((a_t, l_t; \Theta_t)\).

Let:

\[ c_t^* \equiv c^*(a_t, l_t; \Theta_t), \]
\[ u_t^* \equiv u^*(a_t, l_t; \Theta_t). \]
The Value Function

State variables of the household’s problem are \((a_t, l_t; \Theta_t)\).

Let:
\[
c_t^* \equiv c^*(a_t, l_t; \Theta_t),
\]
\[
u_t^* \equiv u^*(a_t, l_t; \Theta_t).
\]

Value function, Bellman equation:
\[
V(a_t, l_t; \Theta_t) = U(c_t^*) - V(l_t + u_t^*) + \beta E_t V(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}),
\]
where:
\[
a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*,
\]
\[
l_{t+1}^* \equiv (1 - s)l_t + f(\Theta_t)u_t^*.
\]
Assumption 1. The function $U(c_t)$ is increasing, twice-differentiable, and strictly concave, and $V(l_t)$ is increasing, twice-differentiable, and strictly convex.

Assumption 2. A solution $V : X \rightarrow \mathbb{R}$ to the household’s generalized Bellman equation exists and is unique, continuous, and concave.

Assumption 3. For any $(a_t, l_t; \Theta_t) \in X$, the household’s optimal choice $(c_t^*, u_t^*)$ exists, is unique, and lies in the interior of $\Gamma(a_t, l_t; \Theta_t)$.

Assumption 4. For any $(a_t, l_t; \Theta_t)$ in the interior of $X$, the second derivative of $V$ with respect to its first argument, $V_{11}(a_t, l_t; \Theta_t)$, exists.
Assumptions about the Economic Environment

**Assumption 5.** The household is infinitesimal.

**Assumption 6.** The household is representative.

**Assumption 7.** The model has a nonstochastic steady state, $x_t = x_{t+k}$ for $k = 1, 2, \ldots$, and $x \in \{c, u, l, a, w, r, d, \Theta\}$. 
Assumptions about the Economic Environment

**Assumption 5.** The household is infinitesimal.

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**Assumption 7.** The model has a nonstochastic steady state, \( x_t = x_{t+k} \) for \( k = 1, 2, \ldots \), and \( x \in \{c, u, l, a, w, r, d, \Theta\} \).

**Assumption 7’.** The model has a balanced growth path that can be renormalized to a nonstochastic steady state after a suitable change of variables.
Arrow-Pratt in a Static One-Good Model (Review)

Compare:

\[ E u(c + \sigma \varepsilon) \; \text{vs.} \; u(c - \mu) \]
Arrow-Pratt in a Static One-Good Model (Review)

Compare:

\[ E \ u(c + \sigma \varepsilon) \ vs. \ u(c - \mu) \]

Compute:

\[ u(c - \mu) \approx u(c) - \mu u'(c), \]
Arrow-Pratt in a Static One-Good Model (Review)

Compare:

\[ E u(c + \sigma \varepsilon) \text{ vs. } u(c - \mu) \]

Compute:

\[ u(c - \mu) \approx u(c) - \mu u'(c), \]

\[ E u(c + \sigma \varepsilon) \approx u(c) + u'(c)\sigma E[\varepsilon] + \frac{1}{2} u''(c)\sigma^2 E[\varepsilon^2], \]
Arrow-Pratt in a Static One-Good Model (Review)

Compare:

\[ E \, u(c + \sigma \varepsilon) \quad \text{vs.} \quad u(c - \mu) \]

Compute:

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Compare:

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\[ u(c - \mu) \approx u(c) - \mu u'(c), \]

\[ E u(c + \sigma \varepsilon) \approx u(c) + \frac{1}{2} u''(c) \sigma^2. \]

\[ \mu = \frac{-u''(c)}{u'(c)} \frac{\sigma^2}{2}. \]
Compare:

\[ E u(c + \sigma \varepsilon) \quad \text{vs.} \quad u(c - \mu) \]

Compute:

\[ u(c - \mu) \approx u(c) - \mu u'(c), \]

\[ E u(c + \sigma \varepsilon) \approx u(c) + \frac{1}{2} u''(c) \sigma^2. \]

Coefficient of absolute risk aversion is defined to be:

\[ \lim_{\sigma \to 0} \frac{2 \mu(\sigma)}{\sigma^2} = \frac{-u''(c)}{u'(c)}. \]
**Arrow-Pratt in a Dynamic Model**

Consider a one-shot gamble in period $t$:

$$a_{t+1} = (1 + r_t) a_t + w_t + d_t - c_t + \sigma \epsilon_t + 1,$$

Note we cannot easily consider gambles over:

- $a_t$ (state variable, already known at $t$)
- $c_t$ (choice variable)

Note $^*$ is equivalent to gamble over asset returns:

$$a_{t+1} = (1 + r_t + \sigma \tilde{\epsilon}_t + 1) a_t + w_t + d_t - c_t,$$

or income:

$$a_{t+1} = (1 + r_t) a_t + w_t + (d_t + \sigma \epsilon_t + 1) - c_t,$$
Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period $t$:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1}, \quad (*)$$
Arrow-Pratt in a Dynamic Model

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(*)

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$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$
Consider a one-shot gamble in period $t$:

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$$a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from $\mu$:

$$\nabla_{1} (a_t, l_t; \Theta_t) \frac{\mu}{(1 + r_t)}$$
Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period $t$:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from $\mu$:

$$\beta E_t \nabla_1 (a^*_{t+1}, l^*_{t+1}; \Theta_{t+1}) \mu.$$
Consider a one-shot gamble in period $t$:

$$a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from $\mu$:

$$\beta E_t \nabla_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \mu.$$

Loss from $\sigma$:

$$\beta E_t \nabla_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \sigma^2 \frac{1}{2}.$$
Coefficient of Absolute Risk Aversion

**Definition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is given by 
\[
R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\frac{\mu(\sigma)}{\sigma^2}.
\]
Coefficient of Absolute Risk Aversion

**Definition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is given by \(R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} \frac{2\mu(\sigma)}{\sigma^2}\).

**Proposition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is well-defined and satisfies

\[
R^a(a_t, l_t; \Theta_t) = -\frac{E_t V_{11}(a^*_t+1, l^*_t+1; \Theta_{t+1})}{E_t V_1(a^*_t+1, l^*_t+1; \Theta_{t+1})}.
\]
Coefficient of Absolute Risk Aversion

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**Proposition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is well-defined and satisfies

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R^a(a_t, l_t; \Theta_t) = \frac{-E_t V_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t V_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}.
\]

**Definition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is given by \(R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2\).

**Proposition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is well-defined and satisfies

\[
R^a(a_t, l_t; \Theta_t) = \frac{-E_t \nabla_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}{E_t \nabla_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}.
\]

Evaluated at the nonstochastic steady state, this simplifies to:

\[
R^a(a, l; \Theta) = \frac{-\nabla_{11}(a, l; \Theta)}{\nabla_1(a, l; \Theta)}
\]

Solve for $V_1$ and $V_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_\tau) - V(l_\tau + u_\tau)]$$

Benveniste-Scheinkman:

$$V_1(a_t, l_t; \Theta_t) = (1 + r_t) U'(c^*_{t})$$

Differentiate $\ast$ to get:

$$V_{11}(a_t, l_t; \Theta_t) = (1 + r_t) U''(c^*_{t}) \frac{\partial c^*_{t}}{\partial a_t}$$
Solve for $\nabla_1$ and $\nabla_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_{\tau}) - V(l_{\tau} + u_{\tau})]$$

Benveniste-Scheinkman:

$$\nabla_1(a_t, l_t; \Theta_t) = (1 + r_t) U'(c_t^*)$$  

($*$)
Solve for $V_1$ and $V_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ U(c_{\tau}) - V(l_{\tau} + u_{\tau}) \right]$$

Benveniste-Scheinkman:

$$\nabla_1(a_t, l_t; \Theta_t) = (1 + r_t) U'(c^*_t). \quad (*)$$

Differentiate (*) to get:

$$\nabla_{11}(a_t, l_t; \Theta_t) = (1 + r_t) U''(c^*_t) \frac{\partial c^*_t}{\partial a_t}.$$
Solve for $\frac{\partial c_t^*}{\partial a_t}$
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Consumption Euler equation:

$$U'(c_t^*) = \beta E_t (1 + r_{t+1}) U'(c_{t+1}^*)$$
Solve for $\partial c_t^*/\partial a_t$

Consumption Euler equation:

$$U'(c_t^*) = \beta E_t (1 + r_{t+1}) U'(c_{t+1}^*),$$

implies, at steady state:

$$\frac{\partial c_t^*}{\partial a_t} = E_t \frac{\partial c_{t+1}^*}{\partial a_t} = E_t \frac{\partial c_{t+k}^*}{\partial a_t}, \quad k = 1, 2, \ldots$$
Solve for \( \frac{\partial c^*_t}{\partial a_t} \)

Consumption Euler equation:

\[
U'(c^*_t) = \beta E_t (1 + r_{t+1}) U'(c^*_{t+1}),
\]

implies, at steady state:

\[
\frac{\partial c^*_t}{\partial a_t} = E_t \frac{\partial c^*_t}{\partial a_t} = E_t \frac{\partial c^*_{t+k}}{\partial a_t}, \quad k = 1, 2, \ldots
\]

Household’s budget constraint, no-Ponzi condition imply:

\[
\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} E_t \left[ \frac{\partial c^*_{t+k}}{\partial a_t} - w \frac{\partial l^*_{t+k}}{\partial a_t} \right] = 1 + r.
\]
Solve for $\frac{\partial l^*_{t+k}}{\partial a_t}$

Labor search Euler equation:

$$\frac{V'(l_t + u_t^*)}{f(\Theta_t)} = \beta E_t \left[ w_{t+1} U'(c_{t+1}^*) - V'(l_{t+1}^* + u_{t+1}^*) ight]$$

$$+ (1 - s) \frac{V'(l_{t+1}^* + u_{t+1}^*)}{f(\Theta_{t+1})}$$

where $\gamma \equiv -\frac{cU''(c)}{U'(c)}$, $\chi \equiv \frac{(l_t + u_t)V''(l_t + u_t)}{V'(l_t + u_t)}$. 
Solve for $\partial l^*_t / \partial a_t$

Labor search Euler equation:

$$\frac{V'(l_t + u^*_t)}{f(\Theta_t)} = \beta E_t \left[ w_{t+1} U'(c^*_{t+1}) - V'(l^*_{t+1} + u^*_{t+1}) ight]$$

$$+ (1 - s) \frac{V'(l^*_{t+1} + u^*_{t+1})}{f(\Theta_{t+1})}$$

and transition equation

$$l_{t+1} = (1 - s)l_t + f(\Theta_t)u_t$$
Solve for $\frac{\partial l_{t+k}^*}{\partial a_t}$

Labor search Euler equation:

$$\frac{V'(l_t + u_t^*)}{f(\Theta_t)} = \beta E_t \left[ w_{t+1} U'(c_{t+1}^*) - V'(l_{t+1}^* + u_{t+1}^*) + (1 - s) \frac{V'(l_{t+1}^* + u_{t+1}^*)}{f(\Theta_{t+1})} \right]$$

and transition equation

$$l_{t+1} = (1 - s) l_t + f(\Theta_t) u_t$$

imply, at steady state:

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[ 1 - (1 - s - f(\Theta))^k \right] \frac{\partial c_t^*}{\partial a_t}.$$
Solve for $\frac{\partial c^*_t}{\partial a_t}$

Household’s budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} E_t \left[ \frac{\partial c^*_{t+k}}{\partial a_t} - w \frac{\partial l^*_{t+k}}{\partial a_t} \right] = 1 + r ,$$
Solve for $\partial c_t^*/\partial a_t$

Household’s budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} \mathbb{E}_t \left[ \frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r ,$$

Consumption Euler equation:

$$\frac{\partial c_t^*}{\partial a_t} = \mathbb{E}_t \frac{\partial c_{t+1}^*}{\partial a_t} = \mathbb{E}_t \frac{\partial c_{t+k}^*}{\partial a_t} , \quad k = 1, 2, \ldots ,$$
Solve for \( \partial c_t^* / \partial a_t \)

**Household’s budget constraint, no-Ponzi condition:**

\[
\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} E_t \left[ \frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r,
\]

**Consumption Euler equation:**

\[
\frac{\partial c_t^*}{\partial a_t} = E_t \frac{\partial c_{t+1}^*}{\partial a_t} = E_t \frac{\partial c_{t+k}^*}{\partial a_t}, \quad k = 1, 2, \ldots,
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**Labor Euler equation:**

\[
E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[ 1 - (1 - s - f(\Theta))^k \right] \frac{\partial c_t^*}{\partial a_t},
\]
Solve for $\frac{\partial c^*_t}{\partial a_t}$

Household’s budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} E_t \left[ \frac{\partial c^*_{t+k}}{\partial a_t} - w \frac{\partial l^*_{t+k}}{\partial a_t} \right] = 1 + r,$$

Consumption Euler equation:

$$\frac{\partial c^*_t}{\partial a_t} = E_t \frac{\partial c^*_{t+1}}{\partial a_t} = E_t \frac{\partial c^*_{t+k}}{\partial a_t}, \quad k = 1, 2, \ldots,$$

Labor Euler equation:

$$E_t \frac{\partial l^*_{t+k}}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[ 1 - (1 - s - f(\Theta))^k \right] \frac{\partial c^*_t}{\partial a_t},$$

Solution is a “modified Golden Rule”:

$$\frac{\partial c^*_t}{\partial a_t} = \frac{r}{1 + w \frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}.$$
Solve for Coefficient of Absolute Risk Aversion

\[ \nabla_1(a, l; \theta) = (1 + r) U'(c), \]
Solve for Coefficient of Absolute Risk Aversion

\( \nabla_1(a, l; \theta) = (1 + r) U'(c), \)

\( \nabla_{11}(a, l; \theta) = (1 + r) U''(c) \frac{\partial c^*_t}{\partial a_t}, \)
Solve for Coefficient of Absolute Risk Aversion

\[ \nabla_1(a, l; \theta) = (1 + r) U'(c), \]

\[ \nabla_{11}(a, l; \theta) = (1 + r)U''(c) \frac{\partial c^*_t}{\partial a_t}, \]

\[ \frac{\partial c^*_t}{\partial a_t} = \frac{r}{1 + w \frac{\gamma}{c} \frac{l + u}{r + s + f(\Theta)}}. \]
Solve for Coefficient of Absolute Risk Aversion

\[ \nabla_1 (a, l; \theta) = (1 + r) U'(c), \]

\[ \nabla_{11} (a, l; \theta) = (1 + r) U''(c) \frac{\partial c^*_t}{\partial a_t}, \]

\[ \frac{\partial c^*_t}{\partial a_t} = \frac{r}{1 + w \frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}, \]

**Proposition 2.** Given Assumptions 1–7, the household’s coefficient of absolute risk aversion, \( R^a(a_t, l_t; \Theta_t) \), evaluated at steady state, satisfies

\[ R^a(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{r}{1 + w \frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}. \]
Relative Risk Aversion

Compare: \[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1} \]

vs.

\[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t. \]
Relative Risk Aversion

Compare: \[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1} \]

vs.

\[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t. \]

**Definition 2.** The households’ coefficient of relative risk aversion, \( R^c(a_t, l_t; \Theta_t) \equiv A_t R^a(a_t, l_t; \Theta_t) \), where \( A_t \) denotes the household’s financial assets plus present discounted value of labor income.

At steady state, \( A = c/r \), and

\[
R^c(a; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + w \frac{\gamma}{\chi} \frac{l + u f(\Theta)}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}.
\]
Numerical Example

Household period utility function:

$$\frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1 + \chi}$$
Numerical Example

Household period utility function:

\[
\frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1+\chi}
\]

Economy is a simple RBC model with labor market frictions:

- Competitive firms,
- Cobb-Douglas production functions, \( y_t = Z_t k_t^{1-\alpha} l_t^\alpha \)
- AR(1) technology, \( \log Z_{t+1} = \rho_z \log Z_t + \varepsilon_t \)
- Capital accumulation, \( k_{t+1} = (1 - \delta)k_t + y_t - c_t \)
- Labor market frictions, \( l_{t+1} = (1 - s)l_t + h_t \)
Numerical Example

Labor market search:

- Cobb-Douglas matching function, \( h_t = \mu u_t^{1-\eta} v_t^\eta \)
- Wage set by Nash bargaining with equal weights
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Labor market search:
- Cobb-Douglas matching function, \( h_t = \mu u_t^{1-\eta} v_t^\eta \)
- Wage set by Nash bargaining with equal weights

Equity security:
- Equity is a consumption claim
- Equity premium is expected excess return,

\[
\psi_t \equiv \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)
\]
Numerical Example

Labor market search:
- Cobb-Douglas matching function, \( h_t = \mu u_t^{1-\eta} v_t^\eta \)
- Wage set by Nash bargaining with equal weights

Equity security:
- Equity is a consumption claim
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\psi_t \equiv \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)
\]

Baseline calibration:
- Production: \( \alpha = 0.7, \delta = .0028, \rho_z = 0.98, \sigma_\varepsilon = .005 \)
- Matching: \( s = .02, \eta = 0.5, v/u = 0.6, f(\Theta) = 0.28 \)
- Preferences: \( \beta = .996, \gamma = 200, \chi = 200, l + u = 0.3 \)
Figure 1: Risk Aversion and Equity Premium vs. $\chi$
Figure 2: Risk Aversion and Equity Premium vs. $\gamma$

- **Fixed-labor measure of risk aversion (left axis)**
- **Relative risk aversion $R_c$ (left axis)**
- **Equity premium (right axis)**

The graph shows the relationship between the coefficient of relative risk aversion $\gamma$ and the equity premium, with fixed-labor measure of risk aversion and relative risk aversion plotted on the left axis and equity premium on the right axis.
Figure 3: Risk Aversion and Equity Premium vs. $f(\Theta)$

- **Fixed-labor measure of risk aversion (left axis)**
- **Equity premium (right axis)**
- **Relative risk aversion $R^c$ (left axis)**
Proposition 3. Given Assumptions 1–8 and fixed values for the parameters $s$, $\beta$, $\gamma$, and $\chi$, $R^c(a, l; \Theta)$ is decreasing in $l/u$. 
Risk Aversion Is Higher in Recessions

**Proposition 3.** *Given Assumptions 1–8 and fixed values for the parameters \( s, \beta, \gamma, \) and \( \chi, \) \( R^c(a, l; \Theta) \) is decreasing in \( l/u. \)*

Proof:

\[
R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}. 
\]

Using \( sl = f(\Theta)u, \)

\[
R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s(1 + l/u)}{r + s(1 + l/u)}}. 
\]
Proposition 4. Let \( f_1, f_2 : \Omega_\Theta \to [0, 1] \). Given Assumptions 1–8 and fixed values for the parameters \( s, \beta, \gamma, \) and \( \chi \), let \( (a_1, l_1; \Theta_1) \) and \( (a_2, l_2; \Theta_2) \) denote corresponding steady-state values of \( (a_t, l_t; \Theta_t) \). If \( f_1(\Theta_1) < f_2(\Theta_2) \), then \( R^c_1(a_1, l_1; \Theta_1) > R^c_2(a_2, l_2; \Theta_2) \).
Proposition 4. Let \( f_1, f_2 : \Omega_\Theta \to [0, 1] \). Given Assumptions 1–8 and fixed values for the parameters \( s, \beta, \gamma, \) and \( \chi \), let \((a_1, l_1; \Theta_1)\) and \((a_2, l_2; \Theta_2)\) denote corresponding steady-state values of \((a_t, l_t; \Theta_t)\). If \( f_1(\Theta_1) < f_2(\Theta_2) \), then \( R^c_1(a_1, l_1; \Theta_1) > R^c_2(a_2, l_2; \Theta_2) \).

Proof:

\[
R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s + f(\Theta)}{r + s + f(\Theta)}}
\]

is decreasing in \( f(\Theta) \).
Two types of households:

- Measure 1 of type 1 households
- Measure 0 of type 2 households
- Type 1 households are more employable: $f_1(\Theta) > f_2(\Theta)$
Two types of households:

- Measure 1 of type 1 households
- Measure 0 of type 2 households
- Type 1 households are more employable: \( f_1(\Theta) > f_2(\Theta) \)

Then Proposition 4 implies \( R^c_2(a_2, l_2; \Theta) > R^c_1(a_1, l_1; \Theta) \).
Table 1: International Comparison

<table>
<thead>
<tr>
<th></th>
<th>s</th>
<th>f(Θ)</th>
<th>percentage of households owning equities</th>
<th>percentage of households owning risky financial assets</th>
<th>share of household portfolios in currency and deposits</th>
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<tbody>
<tr>
<td>United States</td>
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<td>.035</td>
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<tr>
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<td>.007</td>
<td>.033</td>
<td>–</td>
<td>–</td>
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<td>.020</td>
<td>–</td>
<td>–</td>
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<tr>
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### Table 2: International Comparison

<table>
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<th>s</th>
<th>f(Θ)</th>
<th>(s + f(Θ))</th>
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<td>.86</td>
<td>.46</td>
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</table>

**Theoretical labor market benchmarks:**
- **perfect rigidity:** 0 0 0 2 5 10 20
- **near-perfect flexibility:** 1 1 .998 0.86 0.46 2.00 6.68

**International comparison, \(r = .004\):**
- **United States:** .019 .282 .987 0.86 0.46 2.02 6.73
- **United Kingdom:** .009 .056 .942 0.89 0.48 2.10 6.93
- **Germany:** .006 .035 .911 0.90 0.49 2.15 7.09
- **France:** .007 .033 .909 0.90 0.50 2.16 7.10
- **Spain:** .012 .020 .889 0.92 0.51 2.20 7.20
- **Italy:** .004 .013 .810 0.96 0.55 2.36 7.64
Table 2: International Comparison

<table>
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<tr>
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<th>$f(\Theta)$</th>
<th>$\frac{s+f(\Theta)}{r+s+f(\Theta)}$</th>
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# Table 2: International Comparison

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<th>$\gamma = 2$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
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<td>$s$ $f(\Theta)$</td>
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<td>1</td>
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<td>2.00</td>
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<table>
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Table 3: Cyclical Variation in Risk Aversion

<table>
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<th>Relative Risk Aversion $R^c$</th>
<th>$\gamma = 2$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 20$</th>
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</thead>
<tbody>
<tr>
<td>$\chi = 1.5$</td>
<td>$s f(\Theta)$</td>
<td>$s + f(\Theta)$</td>
<td>$r + s + f(\Theta)$</td>
<td>$\chi = 0.5$</td>
<td>$\chi = 2.5$</td>
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<tr>
<td>$r = .004$:</td>
<td>United States, expansion</td>
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<td>0.35</td>
<td>0.989</td>
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<td>United States, recession</td>
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<td>0.20</td>
<td>0.982</td>
<td>0.87</td>
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<td>$r = .0083$:</td>
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<td>0.35</td>
<td>0.978</td>
<td>0.87</td>
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<tr>
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<td>United States, recession</td>
<td>0.022</td>
<td>0.20</td>
<td>0.964</td>
<td>0.88</td>
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<td>$r = .0167$:</td>
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<td>0.35</td>
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<td>0.20</td>
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Conclusions

General conclusions:
- A flexible labor margin affects risk aversion
- Risk premia are closely related to risk aversion

Implications of labor market frictions:
- Risk aversion is higher in recessions
- Risk aversion is higher in more frictional labor markets
- Risk aversion is higher for households that are less employable

Quantitative findings:
- Low discount rate $\Rightarrow$ effects of labor market frictions are small
- Risk aversion formulas in Swanson (2012) a good approximation
- Quantitative effects of frictions can be substantial if discount rate is high (frictions are more costly)