Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Society for Economic Dynamics Meetings, Seoul
June 28, 2013
Suppose a household has preferences:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),
\]

\[
u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta l_t
\]

What is the household’s coefficient of relative risk aversion?
Suppose a household has preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \]

\[ u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta l_t \]

What is the household’s coefficient of relative risk aversion?

Answer: 0
Coefficient of Relative Risk Aversion

Suppose a household has preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \]

\[ u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\chi}}{1+\chi} \]

What is the household’s coefficient of relative risk aversion?

Answer: \( \frac{1}{1 + \frac{1}{\gamma + \chi}} \)
Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):
- Individuals who win a lottery prize reduce labor supply by $.11 for every $1 won (note: spouse may also reduce labor supply)

Coile and Levine (2009):
- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Coronado and Perozek (2003):
- Individuals who held more stocks in late 1990s retired 7 months earlier

Large literature estimating wealth effects on labor supply (e.g., Pencavel 1986)
Frictional Labor Markets

Perfectly rigid labor market:
- Arrow (1964), Pratt (1965), Epstein-Zin (1989), etc.

Perfectly flexible labor market:
- Swanson (2012)

This paper:
- Frictional labor markets
A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_\tau) - V(l_\tau + u_\tau)],$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau)a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T \to \infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

$$\{w_\tau, r_\tau, d_\tau\}$$ are exogenous processes, governed by $$\theta_\tau$$

Labor market search: $$l_{\tau+1} = (1 - s)l_\tau + f(\theta_\tau)u_\tau$$
The Value Function

State variables of the household’s problem are \((a_t, l_t; \theta_t)\).

Let:

\[
\begin{align*}
    c_t^* &\equiv c^*(a_t, l_t; \theta_t), \\
    u_t^* &\equiv u^*(a_t, l_t; \theta_t).
\end{align*}
\]
The Value Function

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Let:

\[
    c_t^* \equiv c^*(a_t, l_t; \theta_t),
\]

\[
    u_t^* \equiv u^*(a_t, l_t; \theta_t).
\]

Value function, Bellman equation:

\[
    W(a_t, l_t; \theta_t) = U(c_t^*) - V(l_t + u_t) + \beta E_t W(a_{t+1}^*, l_{t+1}^*; \theta_{t+1}),
\]

where:

\[
    a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*,
\]

\[
    l_{t+1}^* \equiv (1 - s)l_t + f(\theta_t)u_t^*.
\]
Technical Conditions

**Assumption 1.** The function $U(c_t)$ is increasing, twice-differentiable, and strictly concave, and $V(l_t)$ is increasing, twice-differentiable, and strictly convex.

**Assumption 2.** A solution $W: X \rightarrow \mathbb{R}$ to the household’s generalized Bellman equation exists and is unique, continuous, and concave.

**Assumption 3.** For any $(a_t, l_t; \theta_t) \in X$, the household’s optimal choice $(c_t^*, u_t^*)$ exists, is unique, and lies in the interior of $\Gamma(a_t; \theta_t)$.

**Assumption 4.** For any $(a_t, l_t; \theta_t)$ in the interior of $X$, the second derivative of $W$ with respect to its first argument, $W_{11}(a_t, l_t; \theta_t)$, exists.
Assumptions about the Economic Environment

**Assumption 5.** The household is infinitesimal.

**Assumption 6.** The household is representative.

**Assumption 7.** The model has a nonstochastic steady state, $x_t = x_{t+k}$ for $k = 1, 2, \ldots$, and $x \in \{c, l, a, w, r, d, \theta\}$.

**Assumption 7'.** The model has a balanced growth path that can be renormalized to a nonstochastic steady state after a suitable change of variables.
Arrow-Pratt in a Static One-Good Model

Compare:

\[ E u(c + \sigma \varepsilon) \quad \text{vs.} \quad u(c - \mu) \]
Arrow-Pratt in a Static One-Good Model

Compare:

\[ E u(c + \sigma\varepsilon) \text{ vs. } u(c - \mu) \]

Arrow-Pratt coefficient of absolute risk aversion:

\[ \lim_{\sigma \to 0} \frac{2\mu(\sigma)}{\sigma^2} \]
Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period $t$:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$
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$$a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t - \mu.$$

**Definition 1.** The household’s coefficient of absolute risk aversion at $(a_t, l_t; \theta_t)$ is given by $R^a(a_t, l_t; \theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma) / \sigma^2$. 
Proposition 1. The household’s coefficient of absolute risk aversion at \((a_t, l_t; \theta_t)\), denoted \(R^a(a_t, l_t; \theta_t)\), satisfies

\[
- \frac{E_t W_{11}(a_{t+1}^*, l_{t+1}^*; \theta_{t+1})}{E_t W_1(a_{t+1}^*, l_{t+1}^*; \theta_{t+1})}.
\]
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Evaluated at the nonstochastic steady state, this simplifies to:

\[
R^a(a, l; \theta) = \frac{-W_{11}(a, l; \theta)}{W_1(a, l; \theta)}
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Solve for $W_1$ and $W_{11}$

Benveniste-Scheinkman:

$$W_1(a_t, l_t; \theta_t) = (1 + r_t) U'(c^*_t). \quad (\star)$$
Solve for $W_1$ and $W_{11}$

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$$W_1(a_t, l_t; \theta_t) = (1 + r_t) U'(c_t^*). \tag{\star}$$

Differentiate (\star) to get:

$$W_{11}(a_t, l_t; \theta_t) = (1 + r_t) U''(c_t^*) \frac{\partial c_t^*}{\partial a_t}.$$
Solve for $W_1$ and $W_{11}$

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Differentiate (*) to get:

$$W_{11}(a_t, l_t; \theta_t) = (1 + r_t) U''(c_t^*) \frac{\partial c_t^*}{\partial a_t}.$$

Consumption Euler equation:

$$\frac{\partial c_t^*}{\partial a_t} = \frac{\partial c_{t+1}^*}{\partial a_t} = \frac{\partial c_{t+k}^*}{\partial a_t}, \quad k = 1, 2, \ldots$$
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Labor search Euler equation:

$$\frac{\partial l^*_{t+k}}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\theta)}{s + f(\theta)} \left[ 1 - (1 - s - f(\theta))^k \right] \frac{\partial c^*_t}{\partial a_t}.$$
Solve for $W_1$ and $W_{11}$

Budget constraint:

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w \gamma \frac{l + u}{c} \frac{f(\theta)}{r + s + f(\theta)}}.$$
Solve for $W_1$ and $W_{11}$

Budget constraint:

$$\frac{\partial c^*_t}{\partial a_t} = \frac{r}{1 + w\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\theta)}{r + s + f(\theta)}} .$$

**Proposition 2.** The household’s coefficient of absolute risk aversion in Proposition 1, evaluated at steady state, satisfies:

$$R^a(a; \theta) = \frac{-U''(c)}{U'(c)} \frac{r}{1 + w\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\theta)}{r + s + f(\theta)}} .$$
Relative Risk Aversion

Compare: \[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1} \]

vs.

\[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t. \]
Relative Risk Aversion

Compare: \( a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1} \)

vs.

\( a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t. \)

**Definition 2.** The coefficient of relative risk aversion, \( R^c(a_t; \theta_t) \equiv A^c_t R^a(a_t; \theta_t) \), where \( A^c_t \) denotes the present discounted value of household consumption.

At steady state, \( A^c = c/r \), and

\[
R^c(a; \theta) = \frac{-U''(c)}{U'(c)} \cdot \frac{c}{1 + w \gamma \frac{l + u}{c} \frac{f(\theta)}{r + s + f(\theta)}}.
\]
Household period utility function:
\[
\frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{(l_t + u_t)^{1+\chi}}{1+\chi}, \quad \gamma = 200
\]

Economy is a very simple, standard RBC model:
- Competitive firms
- Cobb-Douglas production, \( y_t = Z_t k_t^{1-\alpha} l^\alpha_t \)
- AR(1) technology, \( \log Z_{t+1} = \rho z \log Z_t + \varepsilon_t \)
- Capital accumulation, \( k_{t+1} = (1 - \delta) k_t + y_t - c_t \)
- Equity is a consumption claim
- Equity premium is expected excess return,
\[
\psi_t = \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)
\]
Figure 1: Risk Aversion and Equity Premium vs. $\chi$

- **Fixed-labor measure of risk aversion (left axis)**
- **Relative risk aversion $R^c$ (left axis)**
- **Equity premium (right axis)**
Figure 2: Risk Aversion and Equity Premium vs. $\gamma$

- **Fixed-labor measure of risk aversion (left axis)**
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Figure 3: Risk Aversion and Equity Premium vs. $f(\theta)$

- **Fixed-labor measure of risk aversion (left axis)**
- **Equity premium (right axis)**
- **Relative risk aversion $R^e$ (left axis)**
Proposition 3. Given Assumptions 1–8 and fixed values for the parameters $s$, $\beta$, $\gamma$, and $\chi$, $R^c(a, l; \theta)$ is decreasing in $l/u$. 
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Proof:

$$R^c(a; \theta) = \frac{-U''(c)}{U'(c)} \cdot \frac{c}{1 + w \frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\theta)}{r + s + f(\theta)}}.$$

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Proof:

$$R^c(a; \theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + w \frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\theta)}{r + s + f(\theta)}}.$$

Using $sl = f(\theta)u$,

$$R^c(a; \theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s(1 + l/u)}{r + s(1 + l/u)}}.$$
Proposition 4. Let $f_1, f_2 : \Theta \rightarrow \mathbb{R}$. Given Assumptions 1–8, let $(a_1, l_1; \theta_1)$ and $(a_2, l_2; \theta_2)$ denote corresponding steady-state values of $(a_t, l_t; \theta_t)$. If $f_1(\theta_1) < f_2(\theta_2)$, then $R^c_1(a_1, l_1; \theta_1) > R^c_2(a_2, l_2; \theta_2)$. 
Proposition 4. Let \( f_1, f_2 : \Theta \rightarrow \mathbb{R} \). Given Assumptions 1–8, let \((a_1, l_1; \theta_1)\) and \((a_2, l_2; \theta_2)\) denote corresponding steady-state values of \((a_t, l_t; \theta_t)\). If \( f_1(\theta_1) < f_2(\theta_2) \), then \( R^c_1(a_1, l_1; \theta_1) > R^c_2(a_2, l_2; \theta_2) \).

Proof:

\[
R^c(a; \theta) = \frac{-U''(c)}{U'(c)} \cdot \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s + f(\theta)}{r + s + f(\theta)}}.
\]
Risk Aversion Higher for Less Employable Households

Two types of households:
- Measure 1 of type 1 households
- Measure 0 of type 2 households
- Type 1 households are more employable: $f_1(\theta) > f_2(\theta)$
Risk Aversion Higher for Less Employable Households

Two types of households:
- Measure 1 of type 1 households
- Measure 0 of type 2 households
- Type 1 households are more employable: $f_1(\theta) > f_2(\theta)$

Then Proposition 4 implies $R^c_2(a, l; \theta) > R^c_1(a, l; \theta)$. 
Table 1: International Comparison

<table>
<thead>
<tr>
<th>Country</th>
<th>s</th>
<th>$f(\theta)$</th>
<th>percentage of households owning equities</th>
<th>percentage of households owning risky financial assets</th>
<th>share of household portfolios in currency and deposits</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>.019</td>
<td>.282</td>
<td>48.9</td>
<td>49.2</td>
<td>12.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>.009</td>
<td>.056</td>
<td>31.5</td>
<td>32.4</td>
<td>26.0</td>
</tr>
<tr>
<td>Germany</td>
<td>.006</td>
<td>.035</td>
<td>18.9</td>
<td>25.1</td>
<td>33.9</td>
</tr>
<tr>
<td>France</td>
<td>.007</td>
<td>.033</td>
<td>–</td>
<td>–</td>
<td>29.1</td>
</tr>
<tr>
<td>Spain</td>
<td>.012</td>
<td>.020</td>
<td>–</td>
<td>–</td>
<td>38.1</td>
</tr>
<tr>
<td>Italy</td>
<td>.004</td>
<td>.013</td>
<td>18.9</td>
<td>22.1</td>
<td>27.9</td>
</tr>
</tbody>
</table>
Table 2: International Comparison

<table>
<thead>
<tr>
<th>Relative Risk Aversion $R^c$</th>
<th>$\gamma = 2$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>$f(\theta)$</td>
<td>$\frac{s+f(\theta)}{r+s+f(\theta)}$</td>
<td>$\chi = 1.5$</td>
<td>$\chi = 0.5$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>.998</td>
<td>.86</td>
<td>.46</td>
</tr>
</tbody>
</table>

Theoretical labor market benchmarks:

- **perfect rigidity**: 0 0 0 2 5 10 20
- **near-perfect flexibility**: 1 1 .998 .86 .46 2.00 6.68

International comparison (based on Hobijn and Şahin, 2007):

- **United States**: .019 .282 .987 0.86 0.46 2.02 6.73
- **United Kingdom**: .009 .056 .942 0.89 0.48 2.10 6.93
- **Germany**: .006 .035 .911 0.90 0.49 2.15 7.09
- **France**: .007 .033 .909 0.90 0.50 2.16 7.10
- **Spain**: .012 .020 .889 0.92 0.51 2.20 7.20
- **Italy**: .004 .013 .810 0.96 0.55 2.36 7.64

Business cycle variation (based on Shimer, 2012):

- **United States, expansion**: .017 .35 .989 0.86 0.46 2.02 6.71
- **United States, recession**: .022 .20 .982 0.87 0.46 2.03 6.75
## Table 3: Higher Costs of Labor Market Frictions

<table>
<thead>
<tr>
<th>Relative Risk Aversion $R^c$</th>
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<tr>
<td>-----------------------------</td>
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<tr>
<td>$r = .004:$</td>
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<td>Italy</td>
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</tr>
<tr>
<td>$r = .012:$</td>
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<td></td>
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<td>United States</td>
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</tr>
<tr>
<td>Italy</td>
<td>.004</td>
<td>.013</td>
<td>.586</td>
<td>1.12</td>
</tr>
</tbody>
</table>
Conclusions

General conclusions:
- A flexible labor margin affects risk aversion
- Risk premia are closely related to risk aversion

Implications of labor market frictions:
- Risk aversion is higher in recessions
- Risk aversion is higher in more frictional labor markets
- Risk aversion is higher for households that are less employable

Quantitative findings:
- Low discount rate $\Rightarrow$ effects of labor market frictions are small
- Risk aversion formulas in Swanson (2012) a good approximation
- Quantitative effects of frictions larger if frictions are more costly