A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt

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Methods and Applications for DSGE Models
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Motivation

Goal: Show that a simple macroeconomic model (with high risk aversion) is consistent with a wide variety of asset pricing facts

- equity premium puzzle
- long-term bond premium puzzle
- credit spread puzzle
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Reduces separate puzzles in finance to a single, unifying puzzle—Why is risk aversion in financial markets so high?
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Reduces separate puzzles in finance to a single, unifying puzzle—Why is risk aversion in financial markets so high?

- financial intermediaries: Adrian-Etula-Muir (2013)
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Secondary theme: Keep the model as simple as possible
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Two key ingredients:

- Epstein-Zin preferences
- nominal rigidities
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Implications for Macroeconomics:
- show how to match risk premia in DSGE framework
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Implications for Macroeconomics:
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Implications for Finance:
- structural model of asset prices (provides intuition, robustness to breaks and policy interventions)
- unifying explanation for asset pricing puzzles
Period utility function:

\[ u(c_t, l_t) \equiv \log c_t - \eta \frac{l_t^{1+\chi}}{1 + \chi} \]

- additive separability between \( c \) and \( l \)
- SDF comparable to finance literature
- log preferences for balanced growth, simplicity
Households

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Flow budget constraint:

\[ a_{t+1} = e^{it}a_t + w_t l_t + d_t - c_t \]
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Calibration: (IES = 1), \( \chi = 2 \), \( l = 1 \) (\( \eta = .54 \))
Generalized Recursive Preferences

Household chooses state-contingent \{ (c_t, l_t) \} to maximize

\[
V(a_t; \theta_t) = \max_{(c_t, l_t)} u(c_t, l_t) + \beta \left( E_t V(a_{t+1}; \theta_{t+1})^{1-\alpha} \right)^{1/(1-\alpha)}
\]

Note: Generalized recursive preferences are often written as:

\[
U(a_t; \theta_t) = \max_{(c_t, l_t)} \left[ \tilde{u}(c_t, l_t) + \beta \left( E_t U(a_{t+1}; \theta_{t+1}) \right)^{\tilde{\alpha}} \right]^{1/(1-\tilde{\alpha})}
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It's easy to map back and forth from \( U \) to \( V \); moreover, \( V \) makes formulas in the paper simpler. \( V \) is more closely related to standard dynamic programming results, regularity conditions, and FOCs.
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Calibration: \( \beta = .99, \ RRA \ (R^c) = 60 \ (\alpha = 80.13) \)
Firms are very standard:
- continuum of monopolistically competitive firms (elasticity $\epsilon$)
- Calvo price setting (probability $1 - \xi$)
- Cobb-Douglas production functions, $y_t(f) = A_t k^{1-\theta} l_t(f)^\theta$
- fixed firm-specific capital stocks $k$

Random walk technology $A_t$
- simplicity
- comparability to Finance
- helps match equity premium

Calibration:
- $\epsilon = 10$
- $\xi = 0.75$
- $\theta = 0.6$
- $\sigma_{A_t} = 0.007$, ($\rho_{A_t} = 1$)
- $k_{Y_4} = 2.5$
Firms

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Calibration: $\epsilon = 10$, $\xi = 0.75$, $\theta = 0.6$, $\sigma_A = .007$, $(\rho_A = 1)$, $\frac{k}{4Y} = 2.5$
Fiscal and Monetary Policy

No government purchases or investment:

\[ C_t = Y_t \]
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Taylor-type monetary policy rule:

\[ i_t = r + \pi_t + \phi_\pi (\pi_t - \bar{\pi}) + \phi_y (y_t - \bar{Y}_t) \]
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Calibration: \(\phi_\pi = 0.5, \phi_y = 0.75, \bar{\pi} = 0.01, \rho_y = 0.9\)
Write equations of the model in recursive form
Solution Method

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Divide nonstationary variables \((Y_t, C_t, w_t, \text{etc.})\) by \(A_t\)
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- second-order: risk premia are constant
- third-order: time-varying risk premia
- higher-order: more accurate over larger region
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Model has 3 state variables \((Y_{t-1}, \Delta_{t-1}, A_{t-1})\) plus 1 shock \((\varepsilon_t)\)
Impulse Responses

Technology $A_t$
Impulse Responses

Consumption $C_t$
Impulse Responses

Inflation $\pi_t$

ann. pct.

10 20 30 40 50

-1.0

-0.8

-0.6

-0.4

-0.2

0.0

pct.

Inflation $\pi_t$
Impulse Responses

Short–term nominal interest rate $i_t$

ann. pct.
Short–term real interest rate $r_t$
Equity: Levered Consumption Claim

Equity price

\[ p_t^e = E_{t+1}(C_{t+1}^\nu + p_{t+1}^e) \]

where \( \nu \) is degree of leverage
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\[ R_{t+1}^e \equiv \frac{C_{t+1}^\nu + p_{t+1}^e}{p_t^e} \]
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Calibration: \( \nu = 3 \)
Table 2: Equity Premium

In the data: 3–6.5 percent per year (e.g., Campbell, 1999, Fama-French, 2002)
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Real Government Debt

Real $n$-period zero-coupon bond price:

\[ p_t^{(n)} = E_t m_{t+1} p_{t+1}^{(n-1)}, \]

\[ p_t^{(0)} = 1, \quad p_t^{(1)} = e^{-r_t} \]
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## Real Yield Curve

Table 3: Real Zero-Coupon Bond Yields

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<tr>
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<th>2-yr.</th>
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Real long-term bonds are like insurance
## Nominal Yield Curve

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\(^a\)Gürkaynak, Sack, and Wright (2007) online dataset
\(^b\)Bank of England web site
# Nominal Yield Curve

## Table 4: Nominal Zero-Coupon Bond Yields

<table>
<thead>
<tr>
<th></th>
<th>1-yr.</th>
<th>2-yr.</th>
<th>3-yr.</th>
<th>5-yr.</th>
<th>7-yr.</th>
<th>10-yr.</th>
<th>(10y)−(1y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Treasuries, 1961–2013&lt;sup&gt;a&lt;/sup&gt;</td>
<td>5.44</td>
<td>5.66</td>
<td>5.84</td>
<td>6.11</td>
<td>6.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Treasuries, 1971–2013&lt;sup&gt;a&lt;/sup&gt;</td>
<td>5.64</td>
<td>5.87</td>
<td>6.07</td>
<td>6.38</td>
<td>6.62</td>
<td>6.89</td>
<td>1.25</td>
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Supply shocks make long-term nominal bonds risky: inflation risk
Nominal Term Premium
Defaultable Debt

Default-free depreciating nominal consol:

\[ p_t^c = E_t m_{t+1} e^{-\pi_{t+1}} (1 + \delta p_{t+1}^c) \]
Defaultable Debt

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\[ i_t^c = \log \left( \frac{1}{p_t^c} + \delta \right) \]
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Nominal consol with default:

\[ p_t^d = E_t m_{t+1} e^{-\pi t+1} \left[ (1 - 1_t^{d t+1})(1 + \delta p_{t+1}^d) + 1_t^{d t+1} \omega_{t+1} p_t^d \right] \]
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The credit spread is \( i^d_t - i^c_t \)
### Table 5: Credit Spread

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<th>average ann. default prob.</th>
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<th>average recovery rate</th>
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Note: (.006) (.58) = 34.8 bp

If default is not cyclical, then it's not risky.

Compare to data: credit spread is about 120 bp (Chen-Collin-Dufresne-Goldstein, 2009; Chen, 2010)
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Compare to data: credit spread is about 120 bp (Chen-Collin-Dufresne-Goldstein, 2009; Chen, 2010)
Default Rate is Countercyclical

Source: Chen (2010)
Recovery Rate is Procyclical

A. Default rates and credit spreads

- Moody's Recovery Rates
  - Altman Recovery Rates
  - Long-Term Mean

source: Chen (2010)
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Discussion

1. Conditional heteroskedasticity
2. First-order vs. second-order stationarity
3. IES ≤ 1 vs. IES ≫ 1
4. Volatility shocks
5. Financial accelerator
Note that

\[ \psi^e_t = - \text{Cov}_t \left( \frac{m_{t+1}}{E_t m_{t+1}}, r_{t+1}^e \right) \]
Endogenous Conditional Heteroskedasticity

Note that

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Risk premium can only vary over time if model implies conditional heteroskedasticity.
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Risk premium can only vary over time if model implies conditional heteroskedasticity

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Here, conditional heteroskedasticity is endogenous

Nonlinear solution contains terms of form

$$x_{t+1}$$

so covariance $\text{Cov}_t$ depends on state $x_t$
Endogenous Conditional Heteroskedasticity

Household period utility function:

\[ u(c_t, l_t) \equiv \log c_t - \eta \frac{l_t^{1+\chi}}{1 + \chi} \]
Endogenous Conditional Heteroskedasticity

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Additive separability with EZ implies model is nonhomogeneous:

Shock to \( A_t, c_t \), causes an additive increase in \( V_t \)

which reduces volatility of

\[ \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \]

and

\[ m_{t+1} = \frac{c_t}{c_{t+1}} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^{-\alpha} \]
First-order vs. Second-order Stationarity

Model has a nonstochastic steady state
And is first-order stable, stationary around steady state
But model is second-order (slightly) nonstationary when $\alpha \neq 0$
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IES and Volatility Shocks

Long-run risks literature typically assumes IES $\gg 1$
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A main motivation: IES $> 1$ implies equity prices fall in response to an increase in volatility
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Extend model above to include volatility shocks:

$$\log \sigma_{A,t} = (1 - \rho_\sigma) \log \bar{\sigma}_A + \rho_\sigma \log \sigma_{A,t-1} + \varepsilon_t^\sigma$$
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Calibration: $\rho_\sigma = .98$, $\text{Var}(\varepsilon_t^\sigma) = (0.1)^2$
Impulse Responses to Volatility Shock

Volatility $\sigma_{A,t}$
Impulse Responses to Volatility Shock

Consumption $C_t$

percent

-0.5
-0.4
-0.3
-0.2
-0.1
0.0

percent

0.0
10 20 30 40 50

Consumption $C_t$
Impulse Responses to Volatility Shock

Inflation $\pi_t$

ann. pct.

0.0

-0.1

-0.2

-0.3

-0.4

-0.5
Impulse Responses to Volatility Shock

Equity premium $\psi_t^e$
Impulse Responses to Volatility Shock

Equity price $p_t^e$

percent

0

10 20 30 40 50

-7

-6

-5

-4

-3

-2

-1

0

percent

10 20 30 40 50
Impulse Responses to Volatility Shock

Nominal term premium $\psi_t$
No Financial Accelerator

With model-implied stochastic discount factor $m_{t+1}$, we can price any asset.

Economy affects $m_{t+1} \Rightarrow$ economy affects asset prices.

However, asset prices have no effect on economy.
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...but not in this paper.
Conclusions

1. A simple macroeconomic model (with high risk aversion) can explain a variety of asset pricing facts/puzzles

2. Unifies asset pricing puzzles into a single puzzle—Why is risk aversion in financial markets so high?

3. Provides a structural framework for intuition about risk premia

4. Suggests a mechanism for feedback from risk premia to macroeconomy