A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt

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Motivation

Goal: Show that a simple macroeconomic model (with high risk aversion) is consistent with a wide variety of asset pricing facts

- equity premium puzzle
- long-term bond premium puzzle (nominal and real)
- credit spread puzzle
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- financial intermediaries: Adrian-Etula-Muir (2013)
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Implications for Macroeconomics:

- show how to match risk premia in DSGE framework
- can endogenize asset price–macroeconomy feedback
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Two key ingredients:
- nominal rigidities
- Epstein-Zin preferences
Households

Period utility function:

\[ u(c_t, l_t) \equiv \log c_t - \eta \frac{l_t^{1+\chi}}{1 + \chi} \]

- additive separability between \( c \) and \( l \)
- log preferences for balanced growth, simplicity
- SDF comparable to finance literature
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Flow budget constraint:

\[ a_{t+1} = e^{it} a_t + w_t l_t + d_t - c_t \]
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$$a_{t+1} = e^t a_t + w_t l_t + d_t - c_t$$

Calibration: (IES = 1), $\chi = 2$, $l = 1$ ($\eta = .54$)
Generalized Recursive Preferences

Household chooses state-contingent \{ (c_t, l_t) \} to maximize

\[
V(a_t; \theta_t) = \max_{(c_t, l_t)} u(c_t, l_t) + \beta \left( E_t V(a_{t+1}; \theta_{t+1})^{1-\alpha} \right)^{1/(1-\alpha)}
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Note: Generalized recursive preferences are often written as:

\[
U(a_t; \theta_t) = \max_{(c_t, l_t)} \left[ \tilde{u}(c_t, l_t)^{\rho} + \beta \left( E_t U(a_{t+1}; \theta_{t+1})^{\alpha} \right)^{\rho/\alpha} \right]^{1/\rho}
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Calibration: \(\beta = 0.99, RRA (\text{R}c) = 60, (\alpha = 80.13)\)
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It’s easy to map back and forth from \(U\) to \(V\); moreover,

- \(V\) makes formulas in the paper simpler
- \(V\) is more closely related to standard dynamic programming results, regularity conditions, and FOCs
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Firms

Firms are very standard:

- continuum of monopolistically competitive firms (elasticity $\epsilon$)
- Calvo price setting (probability $1 - \xi$)
- Cobb-Douglas production functions, $y_t(f) = A_t k^{1-\theta} l_t(f)^\theta$
- fixed firm-specific capital stocks $k$

Random walk technology: $\log A_t = \log A_{t-1} + \varepsilon_t$

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- comparability to Finance
- helps match equity premium
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Calibration: $\epsilon = 10$, $\xi = 0.75$, $\theta = 0.6$, $\sigma_A = .007$, $(\rho_A = 1)$, $\frac{k}{4Y} = 2.5$
Fiscal and Monetary Policy

No government purchases or investment:

\[ C_t = Y_t \]
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Taylor-type monetary policy rule:

\[ i_t = r + \pi_t + \phi_{\pi}(\pi_t - \pi) + \phi_y(y_t - \bar{y}_t) \]
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\[ \bar{y}_t \equiv \rho_{\bar{y}} \bar{y}_{t-1} + (1 - \rho_{\bar{y}}) y_t \]
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Calibration: \(\phi_{\pi} = 0.5, \ \phi_y = 0.75, \ \bar{\pi} = 0.01, \ \rho_y = 0.9\)
Solution Method

Write equations of the model in recursive form
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Divide nonstationary variables \((Y_t, C_t, w_t, \text{etc.})\) by \(A_t\)
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- second-order: risk premia are constant
- third-order: time-varying risk premia
- higher-order: more accurate over larger region
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Model has 3 state variables \((\bar{y}_t, \Delta_t, A_t)\), one shock \((\varepsilon_t)\)
Impulse Responses

Consumption $C_t$

percent

0.0
0.2
0.4
0.6
0.8
1.0

0 10 20 30 40 50

Model

Asset Prices

Discussion

Conclusions
Impulse Responses

Inflation $\pi_t$

_ann. pct._

0.0
-0.2
-0.4
-0.6
-0.8
-1.0

10 20 30 40 50

Inflation $\pi_t$
Impulse Responses

Short-term nominal interest rate $i_t$

-0.5
-0.4
-0.3
-0.2
-0.1
0.0
ann. pct.
Impulse Responses

Short-term real interest rate $r_t$

![Graph showing the short-term real interest rate $r_t$ over time. The graph plots the annual percentage change against time, with a downward curve indicating a decrease in the interest rate over time.](image-url)
Equity: Levered Consumption Claim

Equity price

\[ p_t^e = E_{t}m_{t+1}(C_{t+1}^{\nu} + p_{t+1}^{e}) \]

where \( \nu \) is degree of leverage
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Realized gross return:

\[ R_{t+1}^e \equiv \frac{C_{t+1}^\nu + p_{t+1}^e}{p_t^e} \]
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Calibration: \( \nu = 3 \)
Table 2: Equity Premium

In the data: 3–6.5 percent per year (e.g., Campbell, 1999, Fama-French, 2002)
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Equity Premium

Equity premium $\psi_t^e$

-25
-20
-15
-10
-5
0
ann. bp
Real $n$-period zero-coupon bond price:

$$p_t^{(n)} = E_t m_{t+1} p_{t+1}^{(n-1)},$$

$$p_t^{(0)} = 1, \quad p_t^{(1)} = e^{-r_t}$$
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Real Yield Curve

Table 3: Real Zero-Coupon Bond Yields

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<tr>
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<th>2-yr.</th>
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Real long-term bonds are like insurance
# Nominal Yield Curve

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<sup>a</sup>Gürkaynak, Sack, and Wright (2007) online dataset

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## Nominal Yield Curve

### Table 4: Nominal Zero-Coupon Bond Yields

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Supply shocks make nominal long-term bonds risky: inflation risk
Nominal Term Premium

Nominal term premium $\psi_t^{(40)}$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{nominal_term_premium.png}
\end{figure}
Defaultable Debt

Default-free depreciating nominal consol:

\[ p^c_t = E_t m_{t+1} e^{-\pi_{t+1}} (1 + \delta p^c_{t+1}) \]
Defaultable Debt

Default-free depreciating nominal consol:

\[ p_t^c = E_t m_{t+1} e^{-\pi t} (1 + \delta p_{t+1}^c) \]

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\[ i_t^c = \log \left( \frac{1}{p_t^c} + \delta \right) \]
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The credit spread is \( i_t^d - i_t^c \).
### Table 5: Credit Spread

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Note: \( .006 \times .58 = 34.8 \text{ bp} \)

If default is not cyclical, then it’s not risky.

In the data, credit spread is about 120 bp (Chen-Collin-Dufresne-Goldstein, 2009; Chen, 2010)
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Default Rate is Countercyclical

A. Default rates and credit spreads

- Moody's Recovery Rates
- Altman Recovery Rates

<table>
<thead>
<tr>
<th>Year</th>
<th>Moody's Annual Corporate Default Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920</td>
<td>8%</td>
</tr>
<tr>
<td>1930</td>
<td>6%</td>
</tr>
<tr>
<td>1940</td>
<td>4%</td>
</tr>
<tr>
<td>1950</td>
<td>2%</td>
</tr>
<tr>
<td>1960</td>
<td>0%</td>
</tr>
<tr>
<td>1970</td>
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</tr>
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<td>1980</td>
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</tr>
<tr>
<td>1990</td>
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</tr>
<tr>
<td>2000</td>
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The "Long-Term Mean" recovery rate is 41.4%, based on Moody's data. Shaded areas are NBER-dated recessions.

source: Chen (2010)
Recovery Rate is Procyclical

Figure 1. Default rates, credit spreads, and recovery rates over the business cycle.

Panel A plots the Moody’s annual corporate default rates during 1920 to 2008 and the monthly Baa-Aaa credit spreads during 1920/01 to 2009/02. Panel B plots the average recovery rates during 1982 to 2008. The “Long-Term Mean” recovery rate is 41.4%, based on Moody’s data. Shaded areas are NBER-dated recessions. For annual data, any calendar year with at least 5 months being in a recession as defined by NBER is treated as a recession year.

default component of the average 10-year Baa-Treasury spread in this model rises from 57 to 105 bps, whereas the average optimal market leverage of a Baa-rated firm drops from 50% to 37%, both consistent with the U.S. data.

Figure 1 provides some empirical evidence on the business cycle movements in default rates, credit spreads, and recovery rates. The dashed line in Panel A plots the annual default rates over 1920 to 2008. There are several spikes in the default rates, each coinciding with an NBER recession. The solid line plots the monthly Baa-Aaa credit spreads from January 1920 to February 2009. The spreads shoot up in most recessions, most visibly during the Great Depression, the savings and loan crisis in the early 1980s, and the recent financial crisis in 2008. However, they do not always move in lock-step with default rates (the correlation at an annual frequency is 0.65), which suggests that other factors, such as recovery rates and risk premia, also affect the movements in spreads.

Next, business cycle variation in the recovery rates is evident in this chart. The Moody’s Recovery Rates are shown in solid line, Altman Recovery Rates in dashed line, and Long-Term Mean in dotted line.

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Discussion

1. Endogenous conditional heteroskedasticity
2. First-order vs. second-order stationarity
3. IES $\leq 1$ vs. IES $> 1$
4. Volatility shocks
5. Government purchases and monetary policy shocks
6. Financial accelerator
Note that

$$\psi_t^e = -\text{Cov}_t\left(\frac{m_{t+1}}{E_t m_{t+1}}, r_{t+1}^e\right)$$
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Here, conditional heteroskedasticity is endogenous.

Nonlinear solution contains terms of form

\[ x_t \in t+1 \]

so covariance \( \text{Cov}_t \) depends on state \( x_t \).
Endogenous Conditional Heteroskedasticity

Household period utility function:

\[ u(c_t, l_t) \equiv \log c_t - \eta \frac{l_t^{1+\chi}}{1 + \chi} \]
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\[ \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \]

and

\[ m_{t+1} = \frac{c_t}{c_{t+1}} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^{-\alpha} \]
First-order vs. Second-order Stationarity

Model has a nonstochastic steady state
And is first-order stable, stationary around steady state
But model is second-order (slightly) nonstationary when $\alpha \neq 0$
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Long-run risks literature typically assumes IES > 1, for two reasons:

- ensures equity prices rise in response to an increase in technology
- ensures equity prices fall in response to an increase in volatility
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Impulse Responses to Volatility Shock

Volatility $\sigma_{A,t}$

![Graph showing the response of volatility to a shock over time.](image-url)
Impulse Responses to Volatility Shock

![Graph showing the impulse response of consumption to a volatility shock. The x-axis represents time (in weeks), and the y-axis represents the percent change in consumption. The graph shows a positive trend over time.]
Impulse Responses to Volatility Shock

Inflation $\pi_t$
Impulse Responses to Volatility Shock

Equity premium $\psi_1^e$
Impulse Responses to Volatility Shock

Equity price $p_t^e$

percent

0

10 20 30 40 50

-7

-6

-5

-4

-3

-2

-1

0

percent
Impulse Responses to Volatility Shock

Nominal term premium $\psi_t$ $(40)$

ann. bp

0 10 20 30 40 50
0
5
10
15
20
25
30

Nominal term premium $\psi_t$ $(40)$
Rudebusch and Swanson (2012) consider similar model with
- technology shock
- government purchases shock
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All three shocks help the model fit macroeconomic variables

But technology shock is most important (by far) for fitting asset prices:

- technology shock is more persistent
- technology shock makes nominal assets risky
No Financial Accelerator

With model-implied stochastic discount factor $m_{t+1}$, we can price any asset.

Economy affects $m_{t+1} \Rightarrow$ economy affects asset prices.

However, asset prices have no effect on economy.
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...but not in this paper.
Conclusions

1. A simple macroeconomic model (with high risk aversion) can explain a variety of asset pricing facts/puzzles.

2. Unifies asset pricing puzzles into a single puzzle—Why is risk aversion in financial markets so high? (Literature provides good answers to this question)

3. Provides a structural framework for intuition about risk premia.

4. Suggests a mechanism for feedback from risk premia to macroeconomy.